# Ionospheric electron density profile estimation by spectral riometry

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**Abstract**

In the so-called spectral riometry, cosmic radio noise absorption is measured simultaneously at multiple radio wave frequencies, instead of a single band used in the traditional riometry. The main advantage of this approach is a possibility to invert the electron density height profile of the D-region ionosphere based on the frequency dependence of the absorption. However, this inversion turns out to be both nonlinear and highly ill-posed, hence needing some strong prior information on the unknown.

We demonstrate by simulations how much information the riometer measurements have about the electron density profile. Firstly, we simulate spectral riometer measurements of 100 points between 17-55 MHz (Figs. 2–5, dots in the left panels), corresponding to known electron density profiles (Fig. 1). Cauchy-distributed receiver noise is added to the data points (SNR=10 over the whole frequency range). Then we try to reconstruct the original electron density profile based on the simulated data by applying a smoothing prior and using MCMC to sample from the posterior distribution. To make MCMC sampling feasible, we reduce the dimension of the inference problem by concentrating the sampling in the directions of the leading eigenvectors of the prior covariance matrix.

Simulations reveal that the ionospheric electron density profiles can, indeed, be determined with some reasonable accuracy in the altitudes of relatively strong absorption.

## Simulating the spectral riometer data

Refractive index $n$ of the plasma for the radio wave is given by the Appleton equation

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{(Y\sin\alpha)^2}{2(1-X-iZ)} \pm \sqrt{\frac{(Y\sin\alpha)^4}{4(1-X-iZ)^2} + (Y\cos\alpha)^2}}, \tag{1}$$

where the normalised frequencies are

$$X = \frac{\omega_{pe}^2}{\omega^2} = \frac{N_e e^2}{\epsilon_0 m_e \omega^2}, Y = \frac{\omega_{ge}}{\omega} = \frac{eB}{m_e\omega} \text{ and } Z = \frac{\nu_{en}}{\omega}. \tag{2}$$

It can be shown that for a receiver with linear polarisation, the total absorption is

$$A_{\text{model}} = 10 \times \log_{10}\left(\frac{A_{oq} + A_{xq}}{A_o + A_x}\right), \tag{3}$$

where

$$A_{o(q),x(q)} = \exp\left(\frac{2\omega}{c}\int_{\infty}^{0} \Im n_{o(q),x(q)} dz\right). \tag{4}$$

As a summary, the cosmic radio-noise absorption $A_{\text{model}}$ can be evaluated whenever the electron density profile $N_e(h)$, and the corresponding quiet-day profile $N_{eq}(h)$, are known (electron-to-neutral collision frequency $\nu_{en}(h)$, calculated based on the MSIS-neutral atmosphere, and the Earth's magnetic field $\mathbf{B}(h)$, based on a simple dipole approximation, are used here as static background profiles). Here we simulate the spectral riometer data, shown in the left panels of Figs. 2–5, for 4 electron density profiles having some excess ionisation added to the background photoionisation (Fig. 1).

Furthermore, a Cauchy-distributed receiver noise (SNR chosen to be 10) is added to the data points to ensure more realistic simulation.

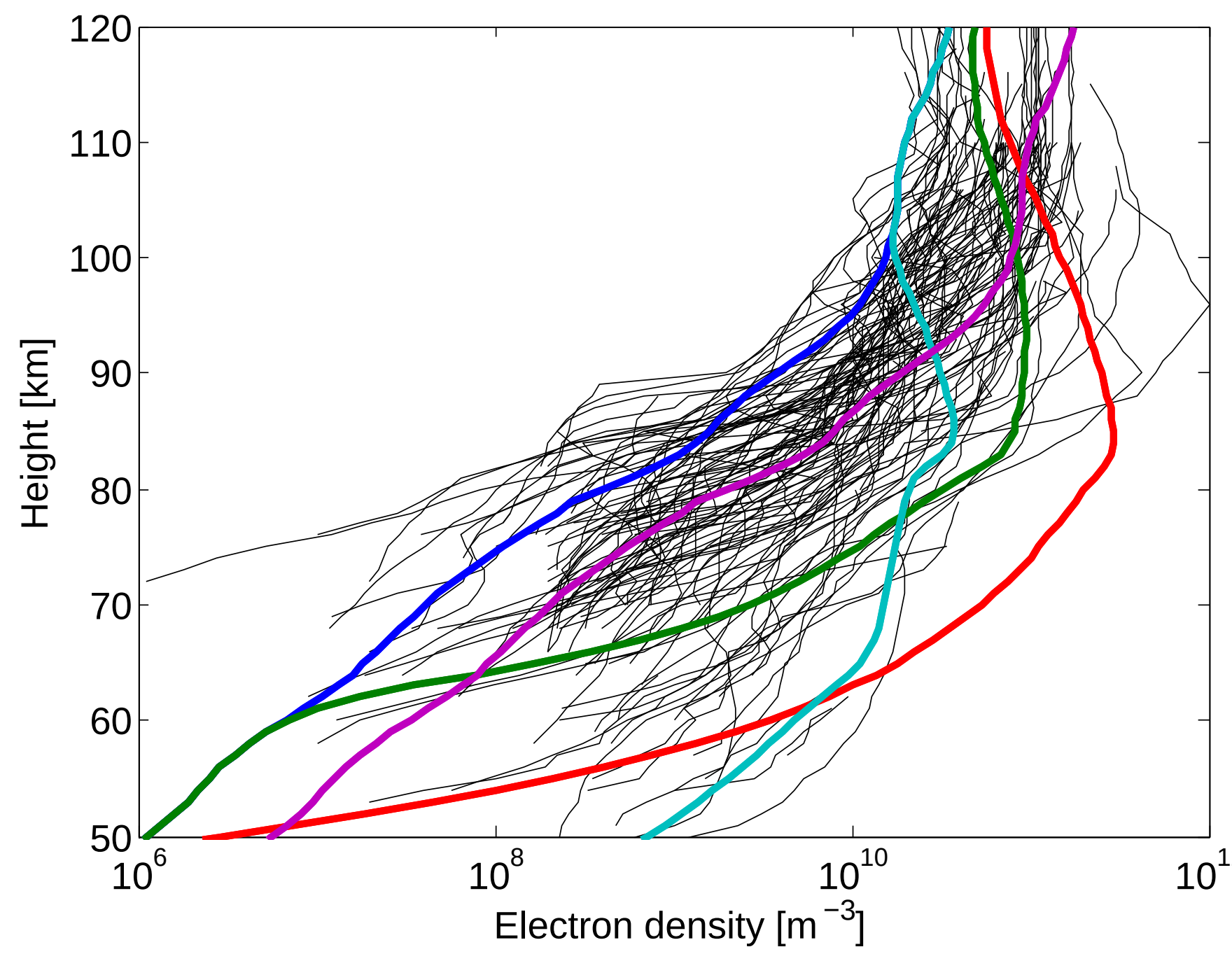

**Figure 1:** Electron density profiles used in the simulations, according to the SIC-model: quiet-day, electron precipitation, hard electron precipitation, Solar proton event, increased photoionsation. A set of high-latitude rocket profiles are plotted in the background (black lines).

## Inversion of the electron density profile

The task is to invert the original electron density profiles based on the simulated data. It turns out, however, that the inversion will be both nonlinear and highly ill-posed, hence needing some strong prior information on the unknown. Instead of constraining the problem physically, we assume only some spatial smoothness by taking samples from the Gaussian covariance function

$$C(x_1, x_2) = \theta \exp\left(-\frac{(h_1 - h_2)^2}{2L^2}\right) + \tau, \tag{5}$$

where we have set the scale parameter $\theta = 10$, correlation length: $L = 22$ km and the "nugget" term $\tau = 10^{-8}$. Instead of the full prior, however, it is possible to use only its $k$ first principal components to define the smooth profile. This makes it feasible to search these components (here 10) by the MCMC approach based on the (simulated) data points and the forward model (Eq. 3). The distributions of results representing the converged MCMC chains (N=10 000) are shown in the Figs. 2–5 for the 4 example profiles studied. The results indicate, that the spectral riometry is capable of reproducing the electron density profile, with reasonable accuracy, at heights of relatively strong absorption.

## Results

In Figures 2–5, the left panels show the simulated spectral riometer data (black dots) calculated for the cases introduced in Fig. 1. The right panels show the corresponding electron density profiles. Distributions representing the converged MCMC chains are shown in both panels (solid black line: median, dark grey: $1\sigma$ and light grey: $2\sigma$ significance levels).

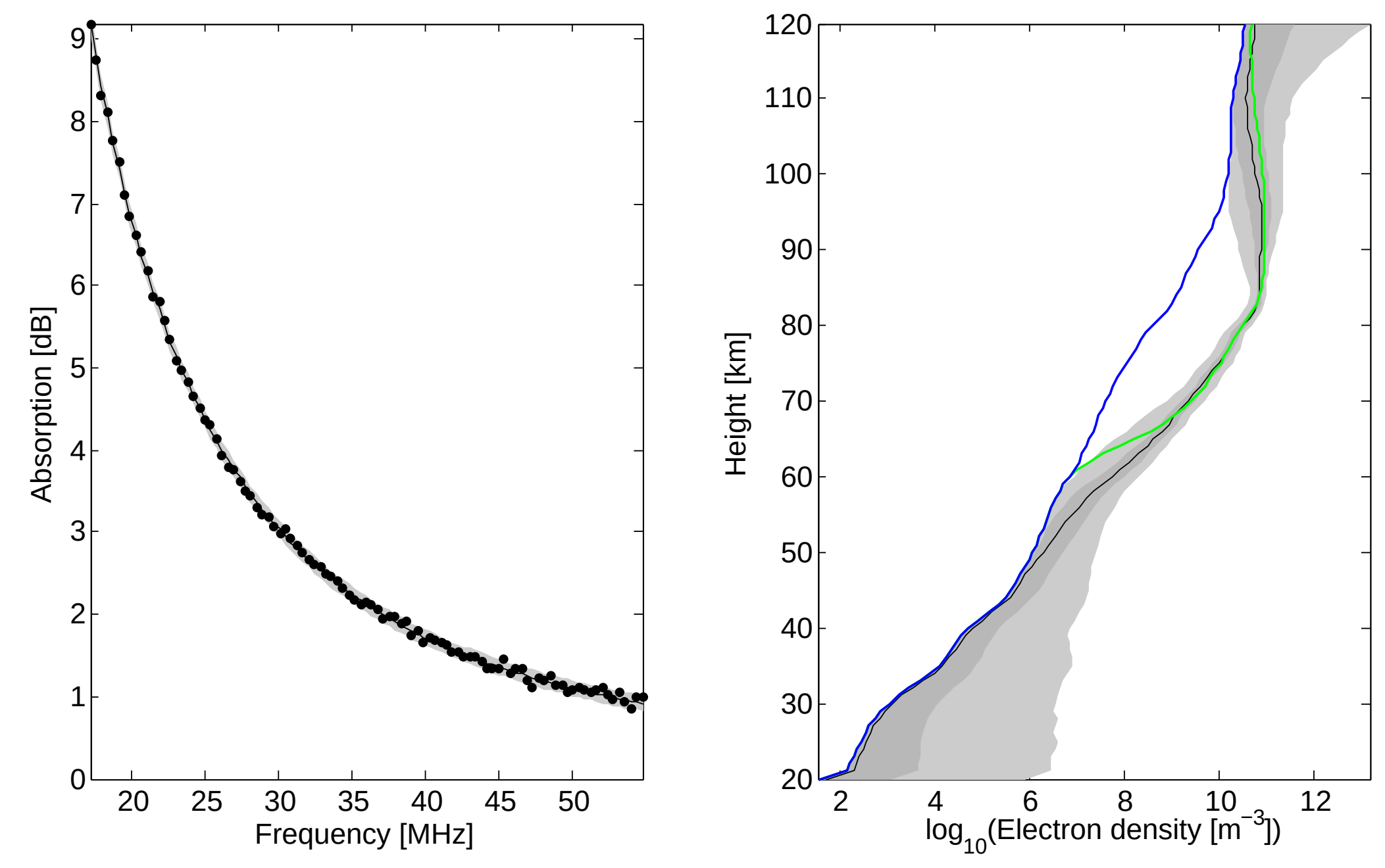

**Figure 2:** Electron precipitation (**green** profile in Fig. 1).

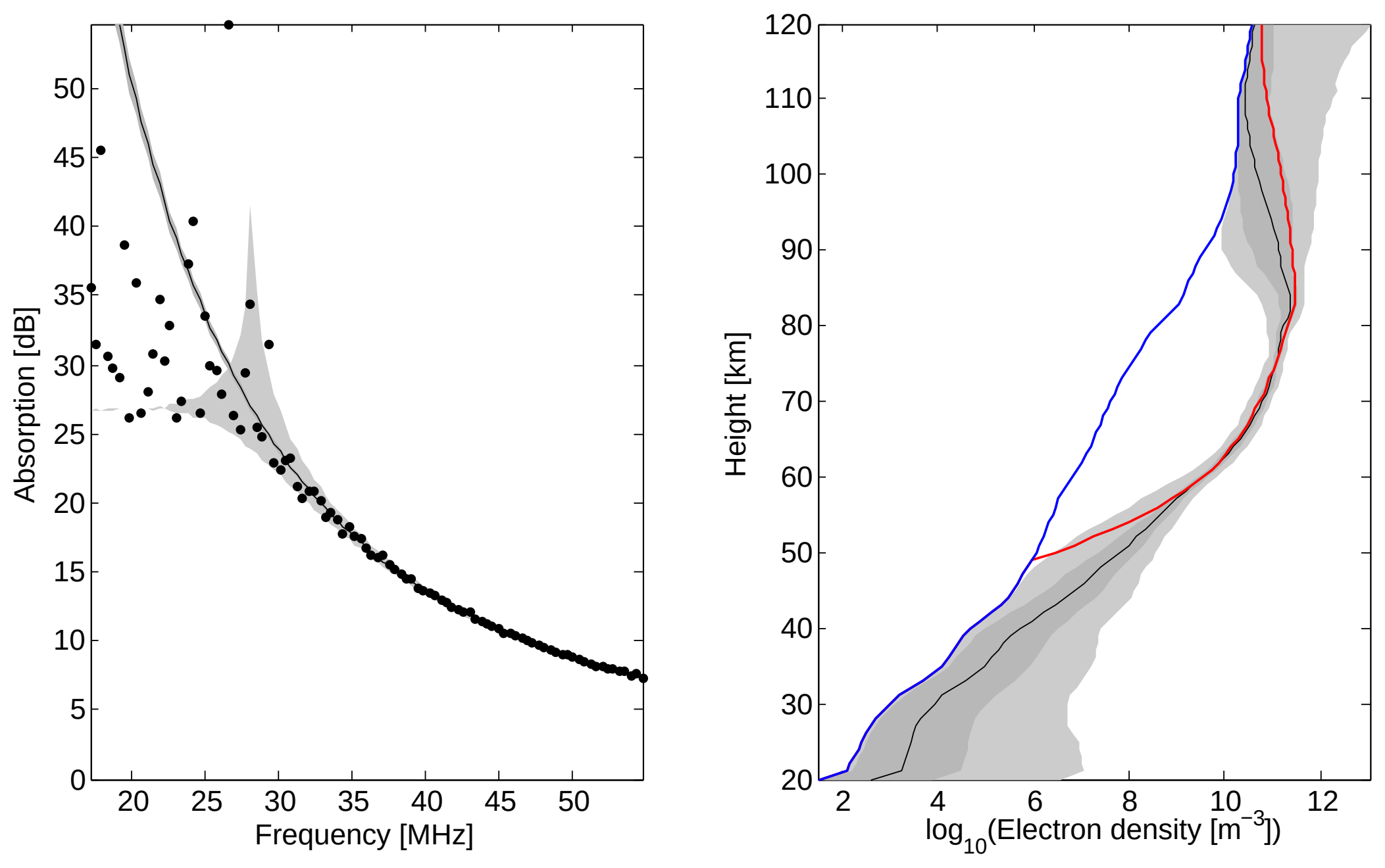

**Figure 3:** Hard electron precipitation (**red** profile in Fig. 1).

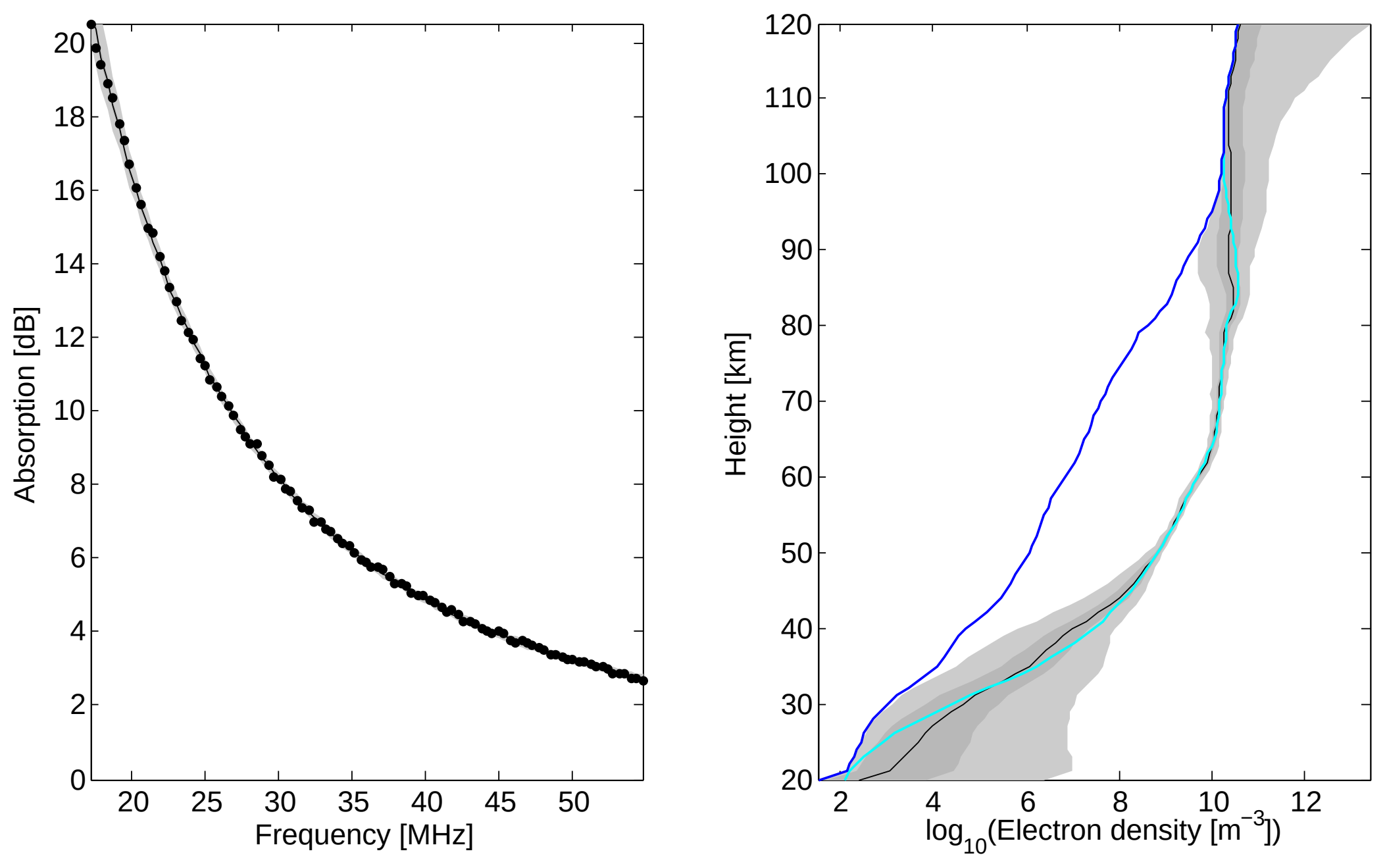

**Figure 4:** Solar proton event (**cyan** profile in Fig. 1).

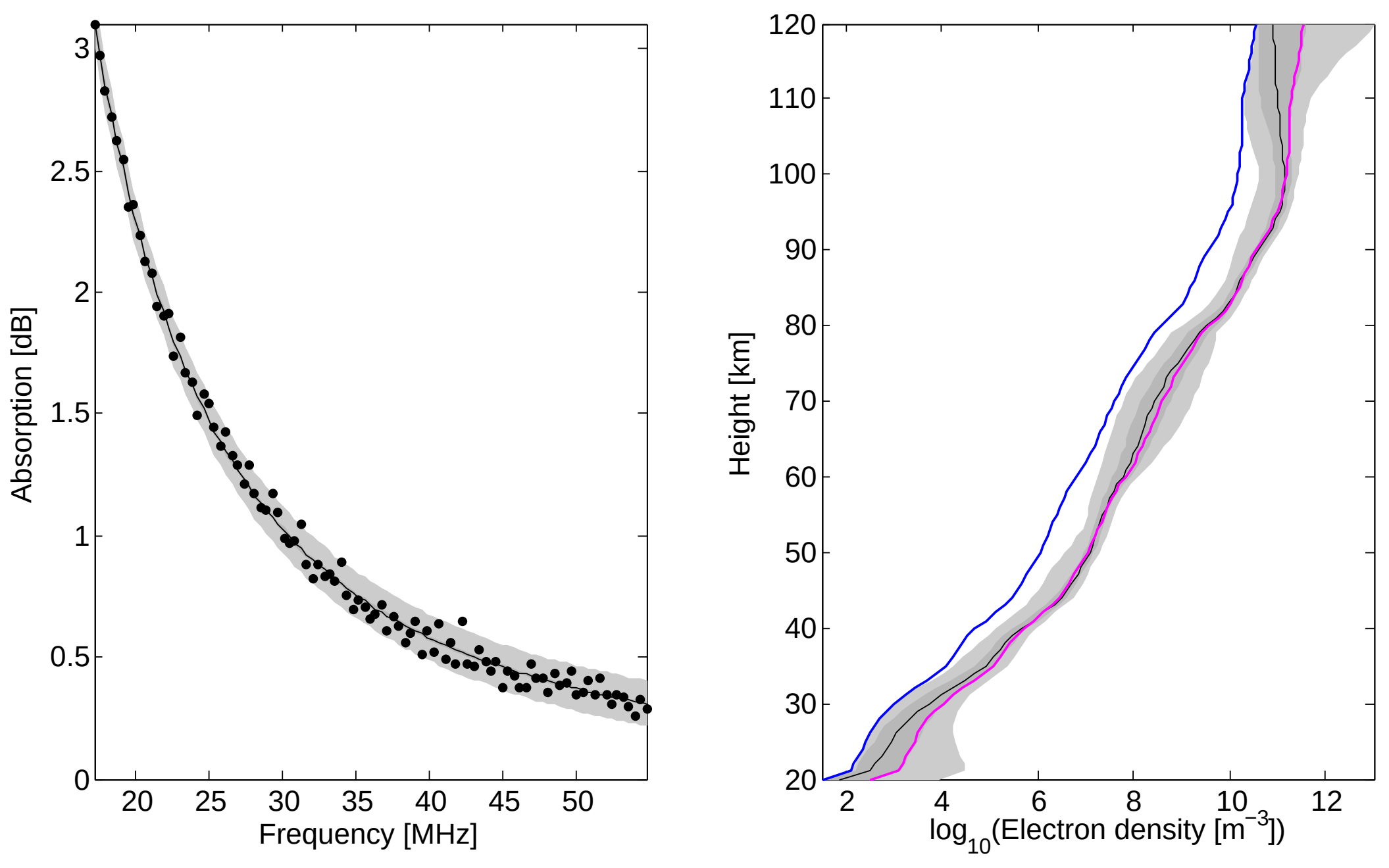

**Figure 5:** Increased photoionisation (**purple** profile in Fig. 1).